



# The interaction between internal heat loss and external heat loss on the extinction of stretched spray flames with nonunity Lewis number

Shuhn-Shyurng Hou \*

*Department of Mechanical Engineering, Kun Shan University of Technology, Tainan, 71003, Taiwan, ROC*

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## Abstract

The influences of flow stretch, preferential diffusion, internal heat transfer and external heat loss on the extinction of dilute spray flames propagating in a stagnation-point flow are analyzed using activation energy asymptotics. A completely prevaporized mode and a partially prevaporized mode of flame propagation are identified. The internal heat transfer, associated with the liquid fuel loading and the initial droplet size of the spray, provides heat loss for rich sprays but heat gain for lean sprays. The flow stretch respectively weakens and intensifies the burning intensity of the lean methanol-spray flame ( $Le > 1$ ) and rich methanol-spray flame ( $Le < 1$ ). Results show that the  $Le > 1$  flame can be extinguished with or without external heat loss. Flame extinction characterized by a C-shaped curve is dominated by the external heat loss or the flow stretch. For the  $Le < 1$  flame without external heat loss, no extinction occurs under the influence of flow stretch. However, the  $Le < 1$  flame with completely prevaporized fuel sprays enduring a small amount of flow stretch can be extinguished by the external heat loss and this behavior is characterized by a C-shaped curve. Note that the W-shaped extinction curve is mainly governed by the internal heat loss. Flame extinction characterized by a W-shaped curve occurs when the  $Le < 1$  spray flame with external heat loss endures a positive stretch and experiences a partially prevaporized spray with sufficiently large liquid fuel loading and droplet size.

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## 1. Introduction

Heat loss is recognized as an extremely important parameter in causing flame extinction. It is well known that a homogeneous laminar premixed flame influenced by external heat loss can be adequately described by a C-shaped extinction curve (a double-valued function) [1–3]. The upper branch of the C-shaped curve shows the stable solutions. It indicates that heat loss decreases the flame velocity and eventually results in flame extinction, identified by the turning point. The lower branch represents the opposite trend, and is generally believed to be

unstable. In addition to heat loss, stretch is a very significant factor affecting flame behavior. The flame in a divergent stagnation-point flow endures a positive stretch, which weakens or strengthens the burning intensity depending on whether the mixture Lewis number is greater or smaller than one [4–7]. Here the Lewis number is defined as the ratio of thermal-to-mass diffusivities of the deficient reactant in the mixture. Flame extinction influenced by stretch is also characterized by a C-shaped extinction curve. Regardless of incomplete reaction and downstream heat loss through the wall, a sufficiently large stretch can result in extinction for  $Le > 1$  flame, while no extinction occurs for  $Le < 1$  flame.

From those studies introduced above, it is clear that flame quenching can be achieved by either external heat loss [1–3] or flow stretch [4–7]. The extinction of

\* Tel.: +886-6-272-4833; fax: +886-6-273-4240.

E-mail address: [sshou@mail.ksut.edu.tw](mailto:sshou@mail.ksut.edu.tw) (S.-S. Hou).

## Nomenclature

### Dimensional quantities

$A'$	cross sectional area of the stream
$B'$	preexponential factor
$C'_{PG}$	specific heat of the gaseous mixture
$C'_{PL}$	specific heat of the liquid
$D'$	mass diffusion coefficient
$E'_a$	activation energy
$L'$	separation distance (Fig. 1)
$\ell'_D$	thickness of the diffusion zone
$\bar{M}'$	average molar mass
$n'$	number density
$Q'$	heat of combustion per unit mass of gaseous fuel
$\tilde{R}$	universal gas constant
$r'$	droplet radius
$S_L^0$	one-dimensional adiabatic flame speed

### Nondimensional quantities

$\bar{A}$	Eq. (9)
$C$	parameter governed by Eq. (29)
$h_{LG}$	latent heat of vaporization, $h'_{LG}/(C'_{PG}T'_i)$
$Le$	Lewis number, $\lambda' / (\rho'_G C'_{PG} D'_i)$
$\dot{m}$	axial mass flux, i.e., burning rate ( $\rho u$ ), $\rho' u' / (\rho'_i S_L^0)$
$P$	pressure, $P' / P'_i$
$Pr$	Prandtl number
$Q$	heat combustion of fuel, $Q' / (C'_{PG} T'_i)$
$q_d$	external heat loss, $q'_d / (C'_{PG} T'_i)$
$T$	temperature, $T' / T'_i$
$T_a$	activation temperature, $E'_a / (\tilde{R} T'_i)$
$T_{ad}$	adiabatic flame temperature, $T'_{ad} / T'_i$
$U$	axial velocity at the burner exit, $u'_i / S_L^0$
$u, v$	axial and radial velocities, $u = u' / S_L^0$ , $v = v' / S_L^0$
$W$	Eq. (10)
$x, y$	axial and radial coordinates, $x = x' / L'$ , $y = y' / L'$
$Y$	mass fraction, $Y_F = Y'_F$ , $Y_O = Y'_O / \sigma$

$z$  density ratio,  $\rho'_G / \rho'$

### Greek symbols

$\alpha$	$\alpha = 1$ and $\alpha = 0$ for lean and rich sprays
$\beta$	mass fraction perturbation in the reaction zone
$\gamma$	$(1 - z_i) / \delta$
$\delta$	small expansion parameter, $\ell'_D / L'$
$\varepsilon$	small expansion parameter, $T'_{ad} \tilde{R} / E'_a$
$\eta$	stretch variable of the reaction zone, $\xi / \varepsilon$
$\theta$	temperature perturbation in the reaction zone
$\lambda'$	thermal conductivity
$\zeta$	stretch variable for the diffusion zone, $(x_f - x) / \delta$
$\rho'$	density
$\sigma$	stoichiometric ratio
$\Phi$	equivalence ratio

### Superscripts

+	downstream near the flame
'	dimensional quantities

### Subscripts

b	boiling state
c	droplet size for completing vaporization just at the flame front
e	state at which droplet is completely gasified
E	flame extinction
f	flame front
F, O	fuel and oxygen
G, L	gas and liquid phases
$i$	state at the exit plane of the burner
$j$	$j = F$ or $O$
$k$	$k = F$ or $O$ in lean and rich mixtures, respectively
s	spray
w	state at wall
0, 1	zeroth- and first-order solutions

homogeneous premixed flames under the influence of a dispersed phase was first reported by Mitani [8] using asymptotic techniques. The application of inert spray [8] or fuel spray [9] effects produced so-called virtual (or internal) heat loss, which is associated with the gasification process of the dispersed phase, and thus led to the S-shaped extinction curve (a triple-valued function) rather than the C-shaped extinction curve. Furthermore, the extinction of a dilute spray flame experiencing external heat transfer, designated by the bulk heat conduction from the system to the surrounding, has been widely analyzed in one-dimensional models [10–12]. It

was concluded that flame quenching is mainly controlled by the external heat loss and is only slightly modified by the internal heat loss associated with the spray effect. However, in these theoretical studies [10–12] the effects of flow stretch and preferential diffusion on the flame extinction of dilute sprays were not examined.

Subsequently, Liu et al. [13] have analyzed the influences of water sprays on the extinction of a methane–air premixed flame propagating in a stagnation-point flow (a two-dimensional model) by using activation energy asymptotics. It was found that flame extinction of the  $Le < 1$  flame by the inert spray is characterized by an

S-shaped curve. Conversely, the  $Le > 1$  flame strongly influenced by flow stretch can be extinguished with or without the participation of the inert spray. Unfortunately, this study did not include the intensified effect of fuel sprays that provide the internal heat gain from the secondary gasified fuel produced by the droplet gasification process under lean-spray conditions [14,15]. Moreover, the interaction between internal heat transfer and external heat loss on stretched spray flames with nonunity Lewis number is of great importance but has not been examined yet.

The objective of this study is to investigate the extinction characteristics under the influences of Lewis number, flow stretch, internal heat transfer and external heat loss. We consider a steady, planar, premixed flame generated in a stagnation-point, two-phase flow in which the dispersed phase is simulated by a monodisperse, dilute and chemically reactive spray. An extinction theory has been formulated on positively stretched spray flames with nonunity Lewis number in a nonconserved system in which the initial gas-phase composition is maintained the same, but the liquid fuel loading is systematically varied. Therefore, the influence of liquid fuel will be independently explored. The heat transfer mechanism is composed of internal and external heat transfer. Here the internal heat transfer, associated with droplet gasification, is a function of the liquid fuel loading and droplet size. The external heat loss is assumed to exist in the downstream region of the premixed flame. The analysis is restricted to dilute sprays, i.e., the amount of liquid fuel loading in the fresh mixture is so small that expansion in perturbation analysis can be performed.

## 2. Formulation

The schematic of the stagnation-point configuration is shown in Fig. 1 in which a two-phase premixture of gaseous fuel, air and liquid fuel droplets impinges onto an impermeable wall. The flow velocity at the exit plane of the burner is assumed to be uniform and is designated by a nondimensional value,  $U$ . The cylindrical coordinates  $(x, y)$  with the origin at the center of the wall are nondimensionalized by the separation distance ( $L'$ ) between the burner and the wall. The axial locations of the burner exit and the premixed flame are respectively denoted by 1 and  $x_f$ . A completely prevaporized mode ( $r'_i \leq r'_c$ ) and a partially prevaporized mode ( $r'_i > r'_c$ ) are identified by a critical initial droplet size ( $r'_c$ ) for the droplet to achieve complete evaporation at the premixed flame front. The droplet starts to evaporate only when the gas temperature has reached the boiling point of the liquid. Droplets then ignite upon acrossing the flame, and vanish at  $x = x_c$  upon complete combustion for lean sprays or complete evaporation for rich sprays.

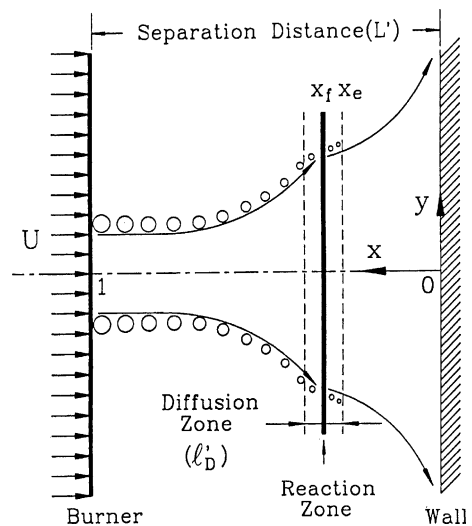


Fig. 1. Schematic diagram of a premixed flame propagating in a stagnation-point flow under the influence of combustible sprays.

Based on large activation energy asymptotics, a small parameter  $\delta = l'_D/L' \ll 1$  is assumed, where  $l'_D = \lambda' / (\rho'_{Gi} C'_{PG} S_L^0)$  indicates the thickness of the diffusion zone. Note that  $\lambda'$  is the thermal conductivity,  $C'_{PG}$  is the specific heat at constant pressure,  $\rho'_{Gi}$  is the density of fresh gas at the burner exit and  $S_L^0$  is the one-dimensional adiabatic laminar flame speed of a premixed flame. According to the fast chemical reaction, a thinner reaction zone is assumed to be embedded within the diffusion zone, as shown in Fig. 1. Since the spray is dilute and monodisperse, the amount of liquid loading is assumed to be  $O(\varepsilon)$  of the total mixture mass. Here the small parameter of expansion,  $\varepsilon$ , is the ratio of thermal energy to large activation energy in the combustion process. In the analysis, small parameters  $\delta$  and  $\varepsilon$  are assumed to be of the same order for the matching of the reaction zone [14]. To illustrate the external heat loss being of  $O(\delta)$  occurs only in the downstream region of the flame. Additionally, moderate rates of flow stretch allowing the flame to sit outside the viscous boundary layer are considered. Finally, the following assumptions are made that the fuel and oxidizer reaction for the bulk premixed flame is one-step overall, that the droplet gasification follows the  $d^2$ -law, and that constant property simplifications apply. More detailed assumptions and comments can be found in earlier studies [13,14].

For a given stream tube of axisymmetrical stagnation-point flow, the total number of droplets crossing any plane normal to the central axis per second is set to be constant,

$$n'u'A' = n'_i u'_i A'_i, \quad (1)$$

where  $A'$  is the cross sectional area of the stream tube,  $n'$  is the number density and  $u'$  is the axial velocity. Following Williams [17], the extent of gas-phase heterogeneity is denoted by the variable  $z = \rho'_G/\rho'$  such that  $z = 1$  represents the completely vaporized state, where  $\rho'$  is the overall density of the two-phase mixture,  $\rho' = \rho'_G + \rho'_S$ . Note that  $\rho'_S = 4/3\pi(r'_i)^3 n' \rho'_L$  shows the spray density. In the formulation, variables are non-dimensionalized by their values at the exit plane of the burner, except that the characteristic velocity for non-dimensionalization is  $S_L^0$  such that  $U = u'/S_L^0$ . Quantities with and without primes are dimensional and non-dimensional, respectively.

Hence, the nondimensional equations for overall continuity, gas-phase continuity, conservations of fuel, oxidizer, energy, and momentum are, respectively, given by

$$\frac{\partial}{\partial x}(\rho u) + \frac{1}{y} \frac{\partial}{\partial y}(y \rho v) = 0, \quad (2)$$

$$\frac{\partial}{\partial x}(\rho z u) + \frac{1}{y} \frac{\partial}{\partial y}(y \rho z v) = \delta^{-1} \bar{A} (1-z)^{1/3} (1-z_i)^{2/3} \times F(T, Y_O)/(zT), \quad (3)$$

$$\begin{aligned} \frac{\partial}{\partial x}(\rho z u Y_j) + \frac{1}{y} \frac{\partial}{\partial y}(y \rho z v Y_j) - \delta Le_j^{-1} \left[ \frac{\partial^2 Y_j}{\partial x^2} + \frac{1}{y} \frac{\partial}{\partial y} \left( y \frac{\partial Y_j}{\partial y} \right) \right] \\ = \delta^{-1} W + f_j \left[ \frac{\partial}{\partial x}(\rho z u) + \frac{1}{y} \frac{\partial}{\partial y}(y \rho z v) \right], \quad j = F, O, \end{aligned} \quad (4)$$

$$\begin{aligned} \frac{\partial}{\partial x}(\rho z u T) + \frac{1}{y} \frac{\partial}{\partial y}(y \rho z v T) - \delta \left[ \frac{\partial^2 T}{\partial x^2} + \frac{1}{y} \frac{\partial}{\partial y} \left( y \frac{\partial T}{\partial y} \right) \right] \\ = -\delta^{-1} W Q + f_T \left[ \frac{\partial}{\partial x}(\rho z u) + \frac{1}{y} \frac{\partial}{\partial y}(y \rho z v) \right] \\ - \delta q_d H(x), \end{aligned} \quad (5)$$

$$\rho \left( u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) = -\bar{P} \frac{\partial P}{\partial x} + \delta Pr \left[ \frac{\partial^2 u}{\partial x^2} + \frac{1}{y} \frac{\partial}{\partial y} \left( y \frac{\partial u}{\partial y} \right) \right], \quad (6)$$

$$\rho \left( u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} \right) = -\bar{P} \frac{\partial P}{\partial y} + \delta Pr \left[ \frac{\partial^2 v}{\partial x^2} + \frac{\partial}{\partial y} \left( \frac{1}{y} \frac{\partial}{\partial y} (y v) \right) \right], \quad (7)$$

where

$$\bar{P} = P'_i / [\rho'_i (S_L^0)^2], \quad (8)$$

$$\bar{A} = 3\ell_D^2 P' \bar{M}' / [T'_i \rho'_L \tilde{R} (r'_i)^2], \quad (9)$$

$$\begin{aligned} W = -(B' \sigma / \bar{M}'_O) (P' \bar{M}' / \tilde{R})^2 \left\{ \lambda' / \left[ C'_{PG} (\rho'_i S_L^0)^2 \right] \right\} Y_F Y_O \\ \times \exp(-T_a/T). \end{aligned} \quad (10)$$

and

$$H(x) = \begin{cases} 1, & x_e \leq x \leq x_f \\ 0, & x > x_f \text{ or } x < x_e. \end{cases} \quad (11)$$

In the above equations,  $Q = Q'/(C'_{PG} T'_i)$  represents the heat of combustion per unit mass of gaseous fuel and  $h_{LG} = h'_{LG}/(C'_{PG} T'_i)$  shows the latent heat of vaporization for the liquid fuel. In Eqs. (3)–(5),  $F(T, Y_O)$ ,  $f_F$ ,  $f_O$ , and  $f_T$  are given by  $\ln[1 + (T - T_b)/h_{LG}]$ , 1, 0, and  $-h_{LG}$  for the vaporizing droplet and  $\ln[1 + (T - T_b + Y_O Q)/h_{LG}]$ , 0,  $-1$ , and  $(Q - h_{LG})$  for the burning droplet, respectively. Finally, the ideal gas equation and the conservation of mass flux for the axisymmetric stream tube are applied to the derivation of the gas-phase continuity.

The boundary conditions at the burner exit,  $x = 1$ , are specified as  $u = U$ ,  $v = 0$ ,  $p = p_i$ ,  $Y_j = Y_{ji}$ ,  $T = T_i$ , and  $z = z_i$ . Since the premixed flame always stays outside of the viscous boundary layer which provides an  $O(\delta^{1/2})$  thick displacement for the outer flow, the boundary conditions at  $x = 0$  can be adequately given by  $u = 0$ ,  $z = 1$ ,  $Y_j = Y_{jw}$ , and  $T = T_w$ . The problem will be separately analyzed by the following three regions, namely the diffusion zone, the reaction zone and the outer hydrodynamic zone. Based on the assumption that  $\delta$  and  $\varepsilon$  are of the same order, the stretched variables are given by  $\xi = (x_f - x)/\delta$  for the diffusion zone and  $\eta = \xi/\varepsilon$  for the reaction zone.

### 3. The diffusion zone

In the diffusion zone, the dependent variables are expanded with respect to the small parameter of  $\delta$  as

$$\begin{aligned} T &= T_0 + \delta T_1 + O(\delta^2), \\ Y_j &= Y_{j0} + \delta Y_{j1} + O(\delta^2), \quad j = F, O, \\ u &= u_0 + \delta u_1 + O(\delta^2), \\ v &= v_0 + \delta v_1 + O(\delta^2), \\ \rho &= \rho_0 + \delta \rho_1 + O(\delta^2) \end{aligned} \quad (12)$$

In order to satisfy the flame structure,  $z$  is also expanded as

$$z = 1 - \delta \gamma z_0 + O(\delta^2) \quad (13)$$

such that  $z_i = 1 - \delta \gamma$  for a dilute spray. The liquid loading is represented by the parameter  $\gamma$ . A larger value of  $\gamma$  means that the dilute spray has a larger amount of liquid fuel. Substituting Eq. (12) into Eqs. (2), (4) and (5), and then expanding yields

$$\begin{aligned} \frac{\partial}{\partial \xi}(\rho_0 u_0) = \frac{\partial}{\partial \xi}(\dot{m}_0) = 0, \\ \dot{m}_0 \frac{\partial Y_{j0}}{\partial \xi} - \frac{1}{Le_j} \frac{\partial^2 Y_{j0}}{\partial \xi^2} = 0, \quad j = F, O, \end{aligned} \quad (14)$$

$$\dot{m}_0 \frac{\partial T_0}{\partial \xi} - \frac{\partial^2 T_0}{\partial \xi^2} = 0,$$

from which the zeroth order solutions are readily determined to be

$$Y_{j0} = \begin{cases} Y_{j,i} - Y_{k,i} e^{\dot{m}_0 L e_j \xi}, & j = F, O, \quad \xi < 0 \\ Y_{j,i} - Y_{k,i} & \xi > 0, \end{cases} \quad (15)$$

and

$$T_0 = \begin{cases} 1 + (T_{ad} - 1) e^{\dot{m}_0 \xi}, & \xi < 0 \\ T_{ad} & \xi > 0, \end{cases} \quad (16)$$

where  $k = F$  and  $k = O$  for lean and rich mixtures, respectively.  $\dot{m}_0$  is a constant and denotes the axial mass flux (burning rate) normalized by the adiabatic premixed value,  $\rho'_i S_L^0$ .

Using Eqs. (13) and (16), we obtain

$$z_0(\xi) = \begin{cases} \left\{ 1 - \frac{2\bar{A}}{3\dot{m}_0} \int_{\xi_v}^{\xi} [1 + (T_{ad} - 1) e^{\dot{m}_0 \xi}]^{-1} \right. \\ \left. \times \ln \left[ 1 + \frac{(1-T_b) + (T_{ad}-1) e^{\dot{m}_0 \xi}}{h_{LG}} \right] d\xi \right\}^{3/2}, & \xi < 0, \\ z_0(0)^{2/3} - \{2\bar{A} \ln[1 + (T_{ad} - T_b + \alpha Q Y_{jw})/h_{LG}]/(3\dot{m}_0 T_{ad})\} \xi, & \xi > 0, \end{cases} \quad (17)$$

from Eq. (3), while the position ( $\xi_v$ ) for the initiation of droplet evaporation is given by

$$\xi_v = \frac{1}{\dot{m}_0} \ln \left( \frac{T_b - 1}{T_{ad} - 1} \right). \quad (18)$$

#### 4. The reaction zone

In the reaction zone of the bulk gas-phase flame, the solution is expanded around the flame-sheet limit as

$$\begin{aligned} T &= T_{ad} + \varepsilon T_{ad} \theta + O(\varepsilon^2), \\ Y_j &= Y_{jf} + \varepsilon \beta_j + O(\varepsilon^2), \quad j = F, O, \end{aligned} \quad (19)$$

to result in

$$\begin{aligned} Le_j^{-1} \frac{d^2 \beta_j}{d^2 \eta^2} &= -T_{ad} \frac{d^2 \theta}{d\eta^2} \\ &= \frac{A}{2} Q \left( \frac{T_{ad}}{T_a} \right) (Y_{Ff} + \varepsilon \beta_F) (Y_{Of} + \varepsilon \beta_O) \exp \theta, \end{aligned} \quad (20)$$

where

$$\begin{aligned} A &= 2 \left( \frac{T_{ad}}{T_a} \right) \left( \frac{B' \sigma}{\bar{M}'_O} \right) \left( \frac{\rho' \bar{M}'}{\bar{R}} \right)^2 \left[ \frac{\lambda'}{C'_{PG} (\rho'_i S_L^0)^2} \right] \\ &\times \exp \left( -\frac{T_a}{T_{ad}} \right) \end{aligned} \quad (21)$$

is the flame speed eigenvalue. By using the local Shvab–Zeldovich formulation and the matching conditions at  $\eta \rightarrow \pm\infty$  [16], we have

$$\dot{m}_0^2 = \exp[T_1(0^+)/T_{ad}] \quad (22)$$

in which the first-order temperature,  $T_1(0^+)$ , denotes the  $O(\delta)$  downstream temperature perturbation at the flame. Eq. (22) shows that the burning rate ( $\dot{m}_0$ ) is affected exponentially by the first-order temperature downstream near the flame. Adding Eq. (4) and (5), and then integrating it from  $\xi = -\infty$  to  $\xi = 0^+$  leads to

$$T_1(0^+) = \gamma \bar{D} - \frac{\bar{K}}{\dot{m}_0^2} - \frac{q_d}{\dot{m}_0^2} \quad (23)$$

in which  $\Gamma = Q Y_{ji} \int_0^1 [1 - \tilde{\omega}^{(Le_j-1)}]/(1 + \tilde{\omega} Q Y_{ji}) d\tilde{\omega}$  represents the Lewis number effect, and  $\bar{K} = (1/y)(\partial/\partial y)(y\omega)$  shows the flow stretch. The spray effect including the liquid fuel loading and the initial droplet size comes from  $\gamma \bar{D}$  in Eq. (23), where

$$\begin{aligned} \bar{D} &= \left[ T_b - \frac{C'_{PL}}{C'_{PG}} (T_b - 1) \right] - (T_{ad} + h_{LG} - \alpha \cdot Q) \\ &\times \left( 1 - \dot{m}_0 \int_0^{\xi_e} z_0 e^{-\dot{m}_0 \xi} d\xi \right) \end{aligned} \quad (24)$$

and

$$\xi_e = z_0(0)^{2/3} / \{2\bar{A} \ln[1 + (T_{ad} - T_b + \alpha Q Y_{jw})/h_{LG}]/(3\dot{m}_0 T_{ad})\}, \quad (25)$$

showing the vaporized state. In addition,  $q_d$  denotes the effect of downstream external heat loss. Here,  $\alpha = 1$  and  $\alpha = 0$  for lean and rich sprays, respectively. The liquid fuel loading is represented by  $\gamma$  through the expansion of  $z_i = 1 - \delta\gamma$  [14]. For the case of completely prevaporized sprays, Eq. (24) is simplified to be

$$\bar{D} = \left[ T_b - \frac{C'_{PL}}{C'_{PG}} (T_b - 1) \right] - (T_{ad} + h_{LG} - \alpha \cdot Q) \quad (26)$$

indicating that there is no contribution on  $T_1(0^+)$  coming from the droplet size.

#### 5. The hydrodynamic zone

The flow stretch, shown in Eq. (23), and the flame position will be determined by solving the flow field of the outer hydrodynamic zone. Since the flame sits outside of the boundary layer and  $\delta \rightarrow 0$ , we neglect the flow viscosity and only consider the zeroth-order solutions that are governed by Eqs. (2), (6) and (7) with boundary conditions being  $u(1) = -U$ ,  $u(0) = v(1) = 0$ . The flow density on either side of the flame is given by

$$\rho = \begin{cases} 1 & x > x_f \\ \rho_w = \rho'_w/\rho'_i & x < x_f. \end{cases} \quad (27)$$

Since the spray effect is being  $O(\varepsilon)$ , jump conditions for the leading order across the flame follow the Rankine–Hugoniot relations [3].

The governing equations admit a self-similar solution of the form

$$u = 2F(x) \quad \text{and} \quad v = -y \frac{d}{dx} F(x). \quad (28)$$

Following the standard procedure used in Kim and Matalon [4] and Tien and Matalon [18], the flow field of the unburnt side of the flame can be found as

$$\begin{aligned} u &= C(x-1)^2 + U(x-2)x, \\ v &= -y(C+U)(x-1), \end{aligned} \quad (29)$$

where  $C$  is a parameter determined by the following equation,

$$\begin{aligned} \frac{1}{2} \rho_w (x_f - 1)^2 \left[ \frac{(x_f - 1)}{\rho_w} - x_f \right]^2 (C + U)^2 \\ + \frac{1}{2} U (C + U) \left[ x_f (x_f - 2) - 2 \frac{(x_f - 1)^2}{\rho_w} \right] \\ + \frac{1}{2} \frac{U^2}{\rho_w} = 0. \end{aligned} \quad (30)$$

For the flow stretch, we find

$$\frac{1}{y} \frac{\partial}{\partial y} (yv) = -2(C+U)(x-1), \quad x > x_f. \quad (31)$$

Since  $\dot{m}_0 = 1u(x_f)$  near the upstream side of the flame, the flame position should be finally identified by the coupling of Eqs. (21), (23), (29), (30) and (31).

Sample calculations based on Eqs. (22)–(26) and (29)–(31) on  $\dot{m}_0$  and  $x_f$  for methanol burning in air are now considered in a nonconserved manner in which the initial gas-phase composition is fixed, i.e.,  $\Phi_G$  is maintained constant, but the liquid fuel loading is systematically varied. Therefore, the influences of liquid fuel will be independently explored without the participation of the leaning effect from the gas-phase mixture. The influences of flow stretch, preferential diffusion and external heat loss on dilute spray flames in the problem will be assessed based on five parameters, namely the initial droplet radius ( $r'_i$ ), the liquid fuel loading ( $\gamma$ ), the flow stretch ( $\bar{K}$ ), the external heat loss ( $q_d$ ) and Lewis number ( $Le$ ). Here  $r'_i$  and  $\gamma$  show the internal heat transfer (heat gain or heat loss) for the fuel spray. We use  $U$  instead of  $\bar{K}$  as a variable to represent flow stretch in the problem because there is a linear relationship between  $U$  and  $\bar{K}$  [13]. Lewis number is defined as  $\lambda' / (\rho'_G C'_{PG} D'_i)$  in which the diffusion coefficient of the deficient reactant in the mixture is used. Methanol-air premixture of  $\Phi_G = 0.8$  ( $Le = 1.0371$ ) and  $\Phi_G = 1.5$  ( $Le = 0.9477$ ) are adopted to illustrate the influence of nonunity Lewis number.

## 6. Lean spray flames with $Le > 1$

Fig. 2 shows the burning rate ( $\dot{m}_0$ ) and the flame position ( $x_f$ ) of lean methanol-spray flames as a function of  $U$  and  $\gamma$  for  $q_d = 0.00$  and  $0.02$  under completely prevaporized conditions ( $r'_i \leq r'_c$ ) in which no liquid droplet exists downstream of the flame. The upper and lower branches of the C-shaped extinction curves respectively denote the stable and unstable solutions. The extinction point, identified by the critical point (represented by the symbol  $\bullet$ ) in connecting the upper and lower branches demonstrates that a sufficiently large flow stretch leads to flame extinction. For the cases of  $q_d = 0.00$  (adiabatic) and  $q_d = 0.02$  (nonadiabatic), for a given  $\gamma$ , the increase in  $U$  first leads to decreases in both  $\dot{m}_0$  and  $x_f$ . This indicates that a weakened flame sits closer to the stagnation plane and suffers a larger flow stretch, and finally reaches flame extinction when the flow stretch is sufficiently large. This characteristic mainly shows the suppression of burning intensity by flow stretch for a  $Le > 1$  flame. For a given  $q_d$ , the lean spray flame with a larger  $\gamma$ , which provides additional internal heat gain resulted from burning the secondary gasified fuel, will be extinguished at a larger flow stretch, as shown in Fig. 2. For a given  $\gamma$ , a lean spray flame with external heat loss ( $q_d = 0.02$ ) will be extinguished at a smaller flow stretch than one without external heat loss ( $q_d = 0.00$ ), as illustrated in Fig. 2. Since both the flow stretch and the external heat loss have negative effects on the  $Le > 1$  flame, it is understood that the extinction of a lean spray flame with  $Le > 1$  is mainly governed by the flow stretch or the external heat loss, and is modified by

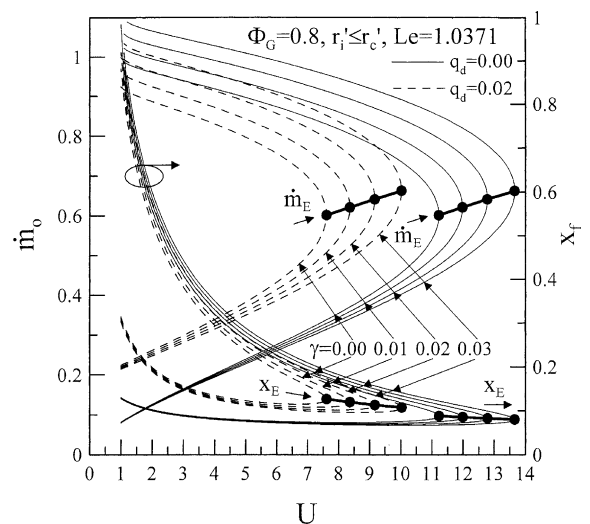


Fig. 2. Variations of  $\dot{m}_0$  and  $x_f$  with  $U$  and  $\gamma$  for lean spray flames with  $q_d = 0.00$  and  $0.02$  under completely prevaporized conditions ( $r'_i \leq r'_c$ ).

the internal heat gain associated with burning the secondary gasified fuel.

Considering the partially prevaporized sprays ( $r'_i > r'_c$ ), the influence of the initial droplet size on flame characteristics is shown in Fig. 3 for a lean methanol-spray flame of  $\Phi_G = 0.8$ ,  $\gamma = 0.02$ , and  $Le = 1.0371$  with  $q_d = 0.00$  and  $0.02$ . Fig. 3 depicts that for both  $q_d = 0.00$  and  $0.02$ , with increasing initial droplet size, the upper branch of the curve corresponding to the stable solution for a partially prevaporized spray first deviates from that for the completely prevaporized spray ( $r'_i \leq r'_c$ ), and approaches that for a homogeneous mixture ( $\gamma = 0$ ). In other words, the burning rate decreases with increasing initial droplet size (due to the reduction of internal heat gain) or flow stretch (caused by the augmentation of the  $Le > 1$  effect). Concerning the droplet gasification process for a lean spray, the liquid fuel absorbs heat for upstream prevaporization, produces the secondary gasified fuel for bulk gas-phase burning, burns through droplet combustion afterwards, and finally leads to internal heat gain. A lean spray containing larger droplets goes through a weaker upstream prevaporization, produces a smaller amount of internal heat gain, and thereby has a decreased burning intensity. Accordingly, it can be extinguished by a smaller flow stretch. Moreover, the extent of extinction point for a lean partially prevaporized spray with  $Le > 1$  is widened when droplet size or external heat loss decreases.

Fig. 4(a) shows the burning rate ( $\dot{m}_0$ ) of lean methanol-spray flames as a function of  $q_d$  and  $\gamma$  at  $U = 2$  for completely prevaporized conditions ( $r'_i \leq r'_c$ ); while Fig.

4(b) shows the burning rate ( $\dot{m}_0$ ) of lean methanol-spray flames as a function of  $q_d$  and  $r'_i$  at  $U = 2$  for partially prevaporized conditions ( $r'_i > r'_c$ ). The C-shaped extinction curves, dominated by external heat loss, are seen in Fig. 4(a) and (b). The upper branches of the C-shaped extinction curves corresponding to stable solutions show that the external heat loss decreases the burning intensity. The lower branches are generally believed to be unstable. The extinction point, identified by the critical point (denoted by the symbol  $\bullet$ ) at which the response curve turns back, indicates that a sufficiently large heat loss results in flame extinction. Fig. 4 also displays that for a lean spray the  $\dot{m}_0$  curve shifts upward with either increasing liquid fuel loading or decreasing droplet size. The amount of heat loss required for the occurrence of flame extinction increases with either increasing  $\gamma$  or decreasing  $r'_i$ . This is owing to the additional heat gain through burning the secondary gasified fuel from the droplet vaporization process for a lean spray.

The burning rate at extinction ( $\dot{m}_E$ ) as a function of  $q_d$  for various values of  $r'_i$  and  $U$  in a lean methanol-spray flame of  $\Phi_G = 0.8$ ,  $\gamma = 0.02$ , and  $Le = 1.0371$  is shown in Fig. 5(a). Results show that  $\dot{m}_E$  decreases with increasing  $r'_i$  or  $U$ , and approaches the asymptotic value,  $\exp(-0.5)$ , identified as the burning rate at extinction of a homogeneous premixture according to the flame quenching theory [3]. It is known that increasing  $r'_i$  suppresses the droplet vaporization and results in a weaker spray burning approaching homogeneous burning. It is also understood that increasing flow stretch ( $U$ ) reduces the burning intensity of a positively stretched

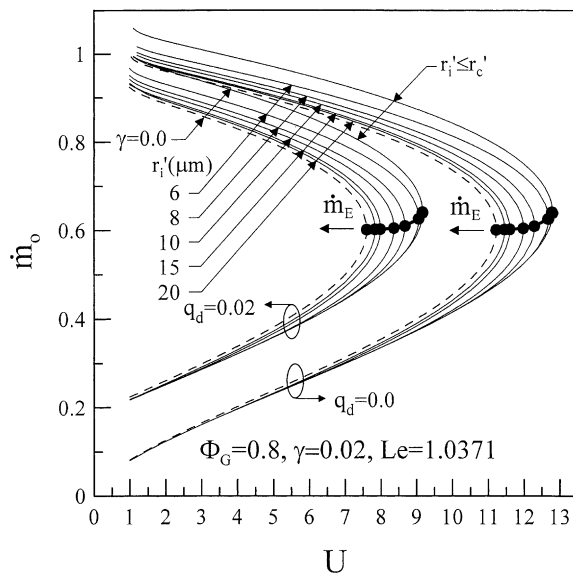


Fig. 3. Variations of  $\dot{m}_0$  with  $U$  and  $r'_i$  for lean spray flames with  $q_d = 0.00$  and  $0.02$  under partially prevaporized conditions ( $r'_i > r'_c$ ).

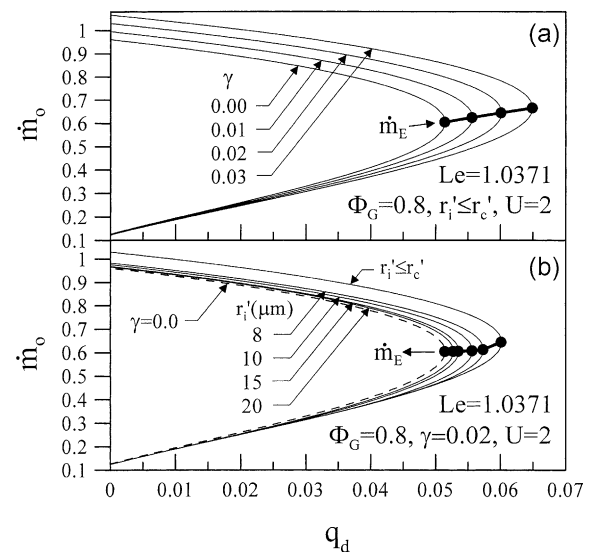


Fig. 4. (a) Variations of  $\dot{m}_0$  with  $q_d$  and  $\gamma$  for lean sprays under completely prevaporized conditions; (b) variations of  $\dot{m}_0$  with  $q_d$  and  $r'_i$  for lean sprays under partially prevaporized conditions.

flame with  $Le > 1$ . Therefore, the value of  $q_d$  at extinction decreases with increasing  $r'_i$  or  $U$ . The burning rate at extinction ( $\dot{m}_E$ ) as a function of  $U$  is demonstrated in Fig. 5(b) for various values of  $r'_i$  and  $q_d$  in lean sprays of  $\Phi_G = 0.8$ ,  $\gamma = 0.02$ , and  $Le = 1.0371$ . It is found that  $\dot{m}_E$  decreases with increasing  $r'_i$  or  $q_d$ , and approaches the asymptotic  $\dot{m}_E$  value of a homogeneous premixture. Increasing  $r'_i$  or  $q_d$  will diminish the burning intensity of the positively stretched lean spray flame with  $Le > 1$ , therefore the value of  $U$  at extinction decreases.

The burning rate at extinction,  $\dot{m}_E$ , as a function of  $U$  is shown in Fig. 5(c) for various values of  $r'_i$  and  $\gamma$  in lean sprays with  $q_d = 0.02$ . It is seen that  $\dot{m}_E$  decreases with increasing  $r'_i$  or decreasing  $\gamma$ , and also approaches the asymptotic value of a homogeneous premixture. Increasing  $r'_i$  or decreasing  $\gamma$  suppresses the droplet vaporization to reduce internal heat gain, and results in a weaker spray burning approaching homogeneous burning. Hence, the value of  $U$  at extinction also decreases with increasing  $r'_i$  or decreasing  $\gamma$ .

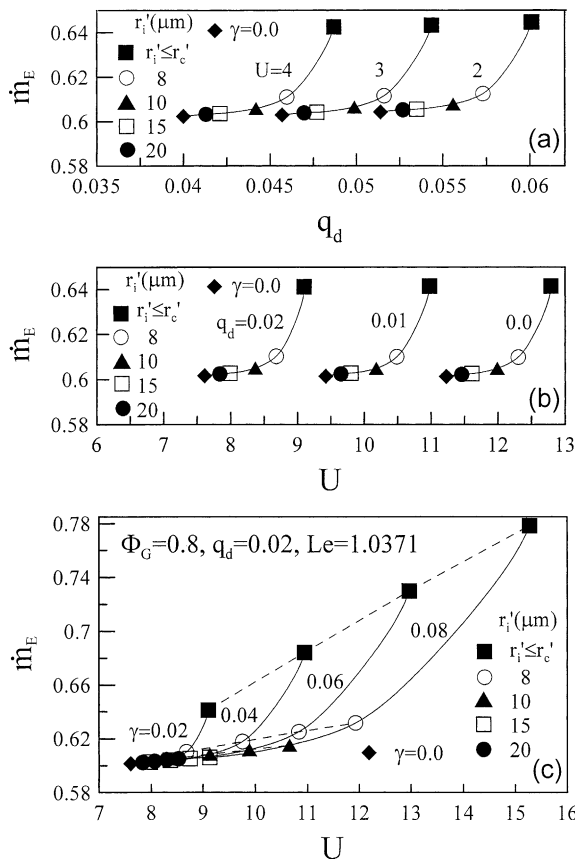


Fig. 5. (a)  $\dot{m}_E$  as a function of  $q_d$  and  $U$  for lean sprays; (b)  $\dot{m}_E$  as a function of  $U$  and  $q_d$  for lean sprays; (c)  $\dot{m}_E$  as a function of  $U$  and  $\gamma$  for lean sprays.

7. Rich spray flames with  $Le < 1$

Fig. 6 shows the burning rate ( $\dot{m}_0$ ) and the flame position ( $x_f$ ) of adiabatic rich methanol-spray flames ( $\Phi_G = 1.5$ ,  $Le = 0.9477$  and  $q_d = 0.0$ ) as a function of  $U$  and  $\gamma$  under completely prevaporized conditions ( $r'_i \leq r'_c$ ). Contrary to the lean spray, the liquid fuel absorbs heat for upstream prevaporization, producing the secondary gasified fuel which is equivalent to an inert substance without any contribution to burning for a rich spray, thus providing an overall internal heat loss, and subsequently weakening the burning rate. For a given  $U$ , an increase in  $\gamma$  (an increase in the internal heat loss) leads to decreases in both  $\dot{m}_0$  and  $x_f$  because a larger  $\gamma$  requires a larger amount of heat absorption from flame to droplets for upstream evaporation. For a given  $\gamma$ , it is found that with increasing flow stretch, the  $Le < 1$  flame is pushed closer to the wall (corresponding to a smaller  $x_f$ ) and has a larger burning rate ( $\dot{m}_0$ ). On the other hand, the decrease of  $U$  first leads to the decrease of  $\dot{m}_0$  and the increase of  $x_f$ , and finally should result in flashback when a smaller value of  $U$  than plotted in the figure is tried. It is noteworthy that no extinction is predicted under the influence of positive stretch at the nondimensional mass flow rate greater than a certain value (characteristic value of the flow system of interest) for the rich methanol sprays with  $q_d = 0.0$ . However, the flow stretch still dominates the flame characteristics.

Fig. 7 shows the burning rate ( $\dot{m}_0$ ) and the flame position ( $x_f$ ) of nonadiabatic rich methanol-spray flames ( $\Phi_G = 1.5$ ,  $Le = 0.9477$  and  $q_d = 0.1$ ) as a function of  $U$  and  $\gamma$  under completely prevaporized conditions. It is seen that for a given  $\gamma$ , the increase of flow stretch results

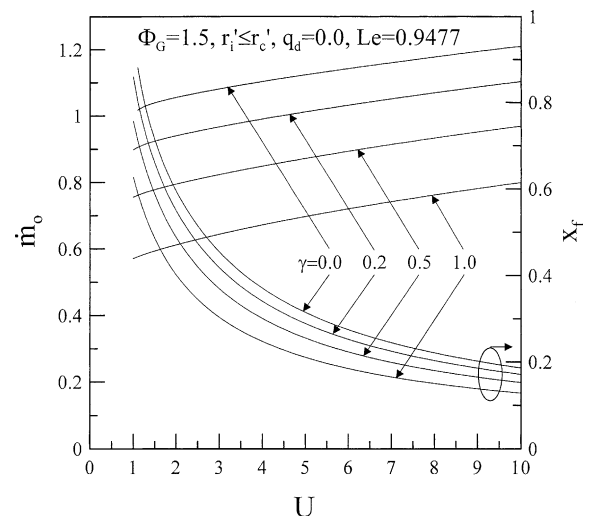


Fig. 6. Variations of  $\dot{m}_0$  and  $x_f$  with  $U$  and  $\gamma$  for rich spray flames with  $q_d = 0.00$  under completely prevaporized conditions.



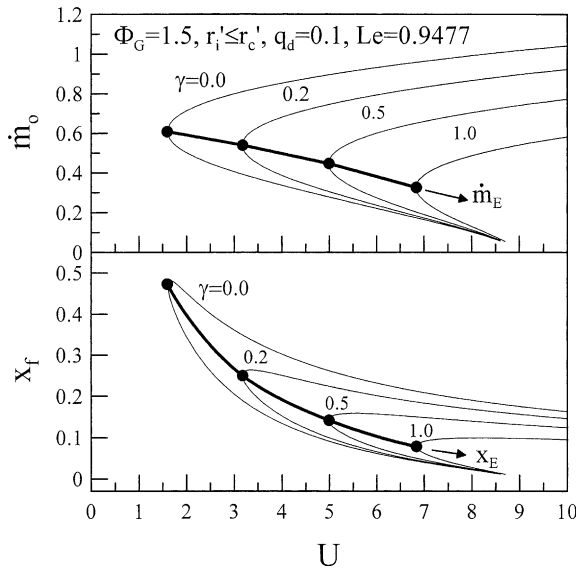


Fig. 7. Variations of  $\dot{m}_0$  and  $x_f$  with  $U$  and  $\gamma$  for rich spray flames with  $q_d = 0.1$  under completely prevaporized conditions.

in the increase of  $\dot{m}_0$  and the decrease of  $x_f$ . However, Fig. 7 further shows that extinction point may be predicted as a turning point on a curve of burning rate versus flow velocity at burner exit before the flash back point occurs. This is due to the external heat loss overwhelming the heat gain by the  $Le < 1$  effect. For a fixed amount of heat loss ( $q_d = 0.1$ ), the  $U$  value at flame extinction increases with  $\gamma$ . Finally, it is noteworthy that the  $Le < 1$  flame with completely prevaporized fuel sprays enduring a small flow stretch can be extinguished by external heat loss and this characteristic is represented by a C-shaped curve.

For partially prevaporized sprays, the response of burning rate ( $\dot{m}_0$ ) and flame position ( $x_f$ ) on the flow stretch is indicated in Fig. 8 with various initial droplet sizes for a rich methanol-spray of  $\Phi_G = 1.5$ ,  $\gamma = 1.5$ ,  $q_d = 0.1$  and  $Le = 0.9477$ . Results show that both the burning rate and the flame position are increased with increasing initial droplet size. This characteristic is due to the reduction of internal heat loss coming from methanol vaporization. The curves for  $r'_i < r'_i^* = 20.7 \mu\text{m}$  in Fig. 8 reveal that the upper branch of the  $\dot{m}_0$  curve controlled by the partially prevaporized sprays merges into that of the  $\dot{m}_0$  curve governed by the completely prevaporized sprays and that flame extinction is achieved at the condition of the flame enduring completely prevaporized sprays. Fig. 8 also shows that the rich-methanol flames ( $Le < 1$ ) experiencing partially prevaporized sprays can be extinguished at a small flow stretch, if the droplet sizes are large enough ( $r'_i \geq r'_i^* = 20.7 \mu\text{m}$ ). The W-shaped extinction curve (a four-valued function) differs from the C-shaped one (a double-valued

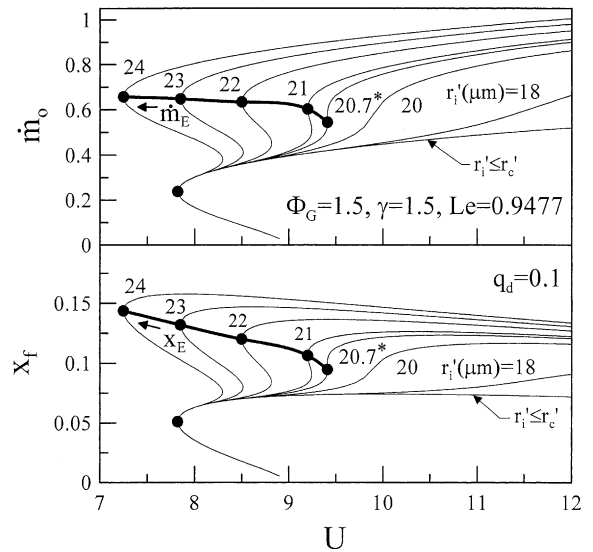


Fig. 8. Variations of  $\dot{m}_0$  and  $x_f$  with  $U$  and  $r'_i$  for rich spray flames with  $q_d = 0.1$ .

function) in the fact that for a  $Le < 1$  spray flame experiencing smaller flow stretch the flame extinction is dominated by internal heat loss. Notably, the burning rate ( $\dot{m}_E$ ) and flame position ( $x_E$ ) at extinction can jump between the C-shaped curve governed by the completely prevaporized sprays and the W-shaped curve controlled by the partially prevaporized sprays. These results suggest that a  $Le < 1$  flame under the influence of a fixed amount of external heat loss can be extinguished by a smaller (larger) flow stretch and a smaller (larger) internal heat loss represented by the rich spray with larger (smaller)  $r'_i$ . Fig. 8 further shows that under the influences of fuel spray, flow stretch and external heat loss, the  $Le < 1$  spray flames can be quenched in the region far away from the wall. These interesting extinction characteristics for the  $Le < 1$  flame were not found in the preliminary study [14].

The response of burning rate ( $\dot{m}_0$ ) and flame position ( $x_f$ ) on the external heat loss ( $q_d$ ) are represented in Fig. 9 for  $\Phi_G = 1.5$ ,  $\gamma = 1.5$ ,  $U = 2$ , and  $Le = 0.9477$  with various initial droplet sizes. It is found that for a given flow stretch ( $U = 2$ ), both  $\dot{m}_0$  and  $x_f$  increase with increasing  $r'_i$  or decreasing  $q_d$ . The former is due to the reduction of internal heat loss associated with the droplet gasification, the latter is caused by the decrease of the external heat loss. For a given value of  $r'_i$  ( $r'_i < r'_i^* = 20.9 \mu\text{m}$ ) Fig. 9 shows that by increasing the external loss from a small value, the burning rate initially influenced by the partially prevaporized spray is monotonically decreased and eventually ends at the completely prevaporized condition. Therefore, flame extinction governed by the completely prevaporized

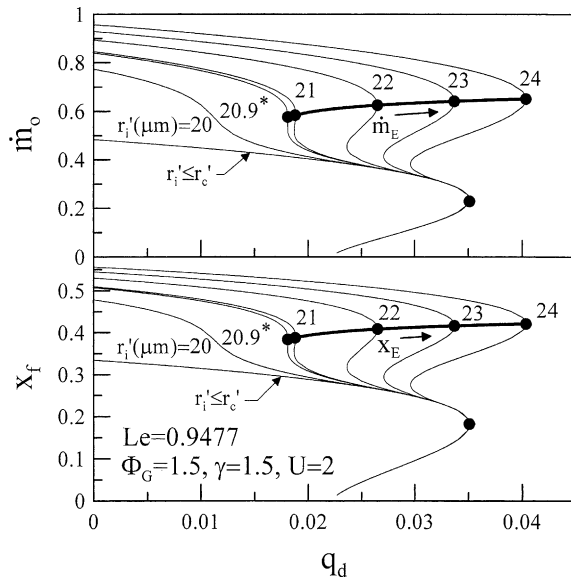


Fig. 9. Variations of  $\dot{m}_0$  and  $x_f$  with  $q_d$  and  $r'_i$  for rich spray flames with  $U = 2$ .

spray occurs if the  $q_d$  is large enough. The curves for  $r'_i \geq r'_i^* = 20.9 \mu\text{m}$  in Fig. 9 display that the  $\dot{m}_0$  curve associated with the partially prevaporized spray is enlarged, and extinction of the  $Le < 1$  flame is therefore dominated by the partially prevaporized spray rather than the completely prevaporized one. Additionally, these curves for  $r'_i \geq r'_i^* = 20.9 \mu\text{m}$  in Fig. 9 demonstrate that with increasing  $r'_i$ , both the burning rate ( $\dot{m}_E$ ) and the corresponding  $q_d$  at extinction are increased, and thus the flame position ( $x_E$ ) at extinction is increased. Notably, we also find that if the droplet size is large enough, the burning rate ( $\dot{m}_E$ ) and flame position ( $x_E$ ) at extinction can jump between the C-shaped curve governed by the completely prevaporized sprays and the W-shaped curve controlled by the partially prevaporized sprays. Results of Fig. 9 suggest that a  $Le < 1$  flame under the influence of a fixed amount of flow stretch can be extinguished by a constant amount of heat loss which may consist of a larger (smaller)  $q_d$  and a larger (smaller)  $r'_i$ . The abrupt change on  $\dot{m}_E$  or  $x_E$  from the completely prevaporized spray to the partially prevaporized spray in Figs. 8 and 9 further emphasizes the significant influence of the droplet size on the flame extinction.

Fig. 10(a) shows the burning rate ( $\dot{m}_0$ ) as a function of the flow stretch ( $U$ ) with different values of  $\gamma$  for a rich methanol-spray flame ( $Le < 1$ ) with  $r'_i = 24 \mu\text{m}$  and  $q_d = 0.1$ . It is known that the flow stretch for the  $Le < 1$  flame is a positive effect on burning intensity. As shown in Fig. 10(a), a large amount of liquid loading (corresponding to a large amount of internal heat loss) leads to a W-shaped extinction curve which gives four solutions for a fixed  $U$ . However, for a small value of  $\gamma$ , the

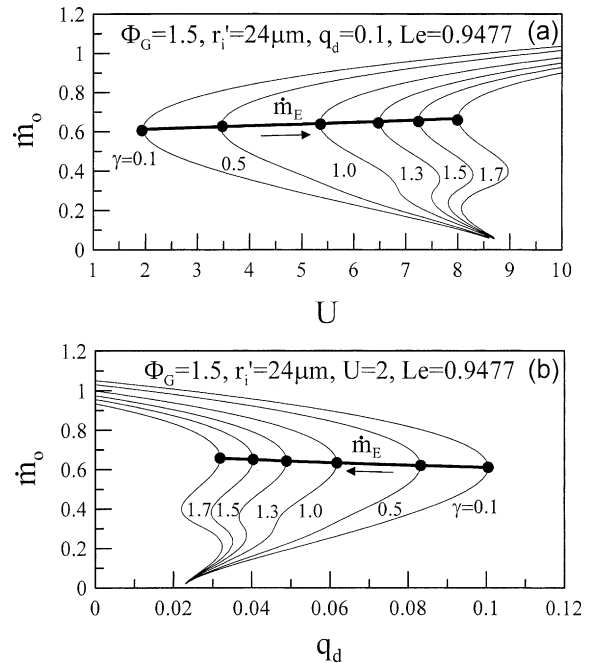


Fig. 10. (a) Variations of  $\dot{m}_0$  with  $U$  and  $\gamma$  for rich sprays with  $r'_i = 24 \mu\text{m}$ ; (b) variations of  $\dot{m}_0$  with  $q_d$  and  $\gamma$  for rich sprays with  $r'_i = 24 \mu\text{m}$ .

C-shaped extinction curve occurs in the region of small flow stretch. On the other hand, as represented in Fig. 10(b), it is found that by setting  $r'_i = 24 \mu\text{m}$  and  $U = 2$ , a large amount of liquid loading also results in a W-shaped extinction curve caused by internal heat loss. Conversely, for a small value of  $\gamma$ , the C-shaped extinction curve exists in the region of large  $q_d$ . These trends in Fig. 10 suggest that a  $Le < 1$  flame under the influence of a fixed amount external heat loss (or flow stretch) can be extinguished by a constant amount of heat loss which may consist of a larger  $\gamma$  corresponding to a larger internal heat loss and a larger flow stretch relating to an augmentation of  $Le < 1$  effect (a smaller  $q_d$  denoting a smaller amount of external heat loss). The W-shaped extinction curve shown in Figs. 8–10 represents the combination of solutions of completely and partially prevaporized spray, and always occurs when the rich methanol-spray flame experiences a partially prevaporized spray composed of a sufficiently large  $r'_i$  and a large enough  $\gamma$ .

The burning rate at extinction,  $\dot{m}_E$ , as a function of  $q_d$  for various values of  $r'_i$  and  $U$  in a rich methanol-spray flame of  $\Phi_G = 1.5$ ,  $\gamma = 1.5$ , and  $Le = 0.9477$  is shown in Fig. 11(a). It is found that the  $\dot{m}_E$  value and its corresponding  $q_d$  at extinction increase with  $r'_i$  or  $U$ . It is expected that increasing  $r'_i$  suppresses the droplet vaporization, leading to the decrease of internal heat loss, and results in a stronger spray burning. It is also known

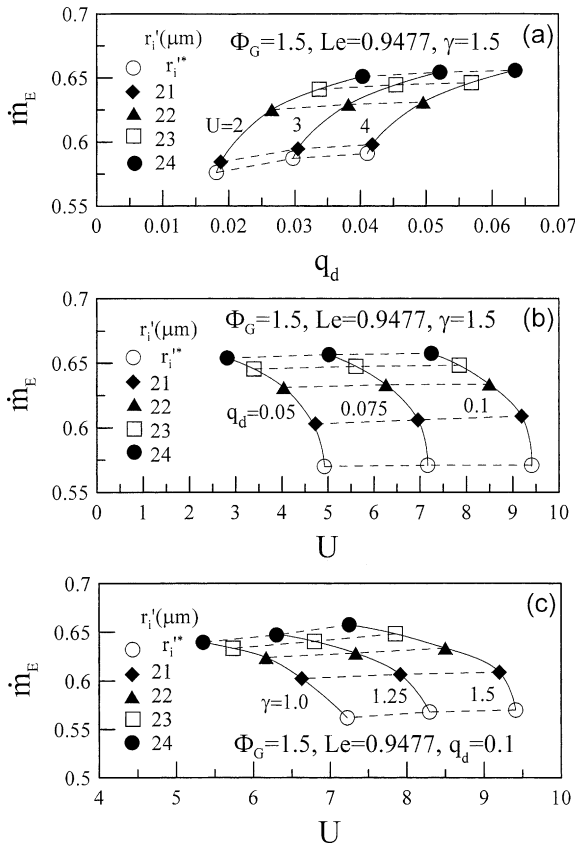


Fig. 11. (a) Variations of  $\dot{m}_E$  with  $q_d$  and  $U$  for rich sprays; (b) variations of  $\dot{m}_E$  with  $U$  and  $q_d$  for rich sprays; (c) variations of  $\dot{m}_E$  with  $U$  and  $\gamma$  for rich sprays.

that increasing flow stretch ( $U$ ) enhances the burning intensity of the positively stretched flame with  $Le < 1$ . Therefore, the value of  $q_d$  at extinction increases with increasing  $r_i'$  or  $U$ . The burning rate at extinction ( $\dot{m}_E$ ) as a function of  $U$  is represented in Fig. 11(b) for various values of  $r_i'$  and  $q_d$  in lean sprays of  $\Phi_G = 1.5$ ,  $\gamma = 1.5$ , and  $Le = 0.9477$ . Since increasing  $r_i'$  (decreasing the internal heat loss) strengthens the burning intensity of the positively stretched rich spray flame with  $Le < 1$ . It is seen that for a fixed value of  $q_d$ , the  $\dot{m}_E$  value increases, while its corresponding  $U$  at extinction decreases with increasing  $r_i'$ . Meanwhile, for a fixed value of  $r_i'$ , both the  $\dot{m}_E$  value and its corresponding  $U$  at extinction increase with  $q_d$ .

As just discussed in Figs. 8–10 that for the rich-methanol spray with  $Le < 1$ , the W-shaped curve occurs when it experiences a partially prevaporized spray with sufficiently large liquid fuel loading and droplet size. The burning rate at extinction of the W-shaped curve,  $\dot{m}_E$ , as a function of  $U$  is shown in Fig. 11(c) for various values of  $r_i'$  and  $\gamma$  in rich sprays with  $q_d = 0.1$ . Results show that for a fixed amount of  $\gamma$ , the  $\dot{m}_E$  value increases,

while its associated flow stretch ( $U$ ) at extinction decreases with increasing initial droplet radius. This characteristic is the same as that described in Fig. 8. It is further found that for a fixed value of  $r_i'$ , the  $\dot{m}_E$  value and its corresponding flow stretch at extinction increase with liquid fuel loading. Considering a rich spray with a fixed value of  $r_i'$ , the increase of  $\gamma$  results in the increase of internal heat loss because the secondary gasified fuel in a rich spray is equivalent to an inert gas with no contribution to burning. Hence, for a fixed value of  $r_i'$ , extinction characterized by a W-shaped curve is achieved at a larger flow stretch when the  $Le < 1$  spray flame contains a larger amount of liquid loading.

## 8. Conclusions

An extinction theory of stretched premixed flames with combustible sprays was developed using activation energy asymptotics to explore the influences of liquid fuel spray, flow stretch, Lewis number and external heat loss on the extinction of methanol spray flames. A completely prevaporized mode and a partially prevaporized mode of flame propagation are identified. The internal heat transfer embedded in the rich and lean spray respectively provides heat loss and heat gain for the system. The flow stretch weakens the burning intensity of the  $Le > 1$  flame (lean methanol-flame), but strengthens that of the  $Le < 1$  flame (rich methanol-flame). Concluding remarks are summarized as follows:

1. For the lean methanol-spray flame with  $Le > 1$ , the burning intensity weakened by the external heat loss or the flow stretch can be intensified when it is composed of a larger amount of liquid fuel loading or a smaller initial droplet size. Note that the  $Le > 1$  flame can be extinguished with or without external heat loss. Both the external heat loss and the flow stretch are found to strongly dominate the tendency for flame extinction characterized by a C-shaped curve.
2. For the rich methanol-spray flame ( $Le < 1$ ) without external heat loss, no extinction is predicted under the influence of positive stretch at the nondimensional mass flow rate greater than a certain value (characteristic value of the flow system of interest). However, the  $Le < 1$  flame with completely prevaporized fuel sprays enduring the flow stretch small enough can be extinguished by the external heat loss characterized by a C-shaped extinction curve.
3. The W-shaped extinction curve (a four-valued function) differs from the C-shaped one (a double-valued function) in the fact that the flame extinction behavior of the former is governed by the internal heat loss. For a  $Le < 1$  spray flame with external heat loss, a W-shaped curve is obtained when it endures a positive stretch and experiences a partially prevaporized spray

with sufficiently large liquid fuel loading and droplet size.

4. For the rich methanol-spray flame ( $Le < 1$ ) with external heat loss, the burning rate ( $\dot{m}_E$ ) and flame position ( $x_E$ ) at extinction can jump between the C-shaped curve (governed by the completely prevaporized sprays) and the W-shaped curve (controlled by the partially prevaporized sprays) in response to varying the droplet size. This interesting characteristic further emphasizes the significant influence of the liquid loading and the droplet size on the flame extinction
5. Flame extinction characterized by a C-shaped curve for the lean methanol-flame with  $Le > 1$  is mainly controlled by the flow stretch or external heat loss and is only slightly modified by the internal heat transfer, while the W-shaped extinction curve for the  $Le < 1$  flame is strongly influenced by the internal heat loss.

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